Measuring Astronomical Masses Name____________________

All astronomical masses are determined by measuring gravity. There is no other way, even for the mass of the earth.

The gravity (and therefore the mass) of an astronomical body can be measured in several ways:

1. Deflection of a moving object. e.g. Asteroid masses deflect spacecraft.
2. Binding of a group of objects. Star cluster masses are measured by this effect.
3. Binding of gas to an object or cluster. Elliptical galaxy masses are sometimes found this way.
4. Orbits of two-body systems. This is the most direct and simplest measure of mass.

We will analyze orbits by appeal to Kepler's laws of planetary motion, as modified by Newton. The laws are:

1. Orbits are ellipses, with the center of mass at one focus.
2. The area swept out by the line from the center of mass to an orbiting body increases at a constant rate.
3. For masses m_1 and m_2, period p, and semi-major axis a: $p^2 = a^3/(m_1+m_2)$
4. We can express the above as $m_1 + m_2 = a^3/p^2$

The reason this 3rd law seems so simple and unit-free is that it is normalized to the solar system. The period p is in years, a is in astronomical units (1 AU = 1.5×10^{11}m), and the masses are in solar masses.

Example: Sun's mass Each solar-system planet forms a two-body system with the sun. This is a good approximation, since the influence of other planets is small. Let's find the mass of the sun, using data from the sun-Jupiter system.
The semi-major axis a = 5.203 AU (The center of mass is inside the sun.)
The orbital period of Jupiter is 11.86 years.

The total mass $m_{sun} + m_{moon} = (5.203)^3/(11.862)^2 = 1.001$ solar masses. The 1 is one solar mass; the additional .001 solar mass is Jupiter's mass.

Exercise: Find the total mass:
1. Sirius system: a=20 AU, p=50 yr Total Mass = __________________

2. Wolf 424 system: a = 4.1 AU, p = 16.2 years Total Mass = __________________

3. 36 Oph system: a = 88 AU, p = 569 years Total Mass = __________________

4. 70 Oph system: a = 23.3 AU, p = 88.3 years Total Mass = __________________

5. EZ Aquarii system: a = 1.20 AU, p = 2.25 years Total Mass = __________________

6. Luyten 726-8 system: a = 5.21AU, p = 26.5 years Total Mass = ___________________

The above analysis, when applied to galaxies and clusters of galaxies, leads to an important concept: dark matter. First, we calculate the mass interior to the orbit of the sun. Visually, that looks like most of the galaxy. Most of the light from a spiral galaxy comes from a region much less than 25,000 lyr from the center. For this calculation:

$a = 1.8 \times 10^9$ AU, $p = 2.5 \times 10^8$ yr Total Mass = ___________________

The standard value for the luminous mass is 10^{11} solar masses, distributed over 2×10^{11} stars.

Modern research has traced the orbital speeds of stars and clouds far beyond the orbit of the sun. The speeds do not obey Kepler's original law, but stay constant out to great distances. They would fall off predictably if the mass were primarily within our orbit. Apparently it is not. Let's calculate the mass interior to a region 5X farther out than the sun:
$a = 9 \times 10^9$ AU (5X greater than above), and p is also 5 X greater since the velocity is the same. The mass is higher by a factor $5^3/5^2 = 5$, yet we see so little mass out there that finding orbits was difficult! The additional 4 galactic masses can't be seen, but is there. It used to be called "missing mass"; it is now called "dark matter".

In the galaxy dark matter is of very low density. The region within 5X our distance from the center ha a volume 125X the volume of the visible part, but only 5X the mass.

The same dark matter is in other galaxies. Clearly our Milky Way galaxy is bound to the Andromeda galaxy; we are approaching one another at 130 km/s. The mass required is at least 5X the combined luminous masses, and probably is greater. In giant clusters of galaxies the motions are random, so the average speeds are important. The mass required to bind these clusters is about 10X the luminous mass. Some clusters have million degree gas trapped in them. The mass required to keep it trapped is the same as calculated above. Dark matter seems to be almost everywhere that galaxies are. Surprisingly, some galaxies don't have any.